Scattering of Internal Gravity Waves at Finite Topography

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LONG-TERM GOAL

The long term goal of the research project is the construction of a numerical model that

- predicts the internal wave field and internal wave induced transports regionally and globally, and

- can be used in conjunction with circulation, turbulence, acoustic and other models for research applications.

OBJECTIVES

The internal wave model will be based on the integration of the radiation balance equation. The current project explores some of the basic issues that arise in the design of such a model:

- Identification of the major dynamical processes that need to be included in the model.
- Assessment of the accuracy of the radiation balance equation, especially in describing the reflection and scattering of internal waves at finite topography.
- Determination of the accuracy of internal wave dispersion relations that are only locally valid and neglect the meridional component of the earth's rotation.
- Utilization of LargeEddy Simulation (LES) models for the study of internal wave breaking and mixing.

APPROACH

The radiation balance equation describes changes of the action density spectrum of the internal wave field along wave group trajectories caused by generation, transfer, and dissipation processes. The predicted quantity is the action density spectrum as a function of wavenumber, position, and time. Overall, the project will emulate the WAM project for surface gravity waves in its approach and methodology.

The radiation balance equation makes three basic approximations:

(1) The random phase approximation.

(2) The WKB or geometric optics approximation.

(3) The weak interaction approximation.

The random phase approximation assumes that the various dynamical processes affecting the internal wave field will distort the wave phases in an irregular way such that it is neither possible nor desirable to predict the wave phases. Instead the wave field is described by its energy or action density spectrum.

The WKB approximation assumes that the wavelengths of the waves are small compared to the scales of the environment. The action density spectrum as a function of wavenumber then varies slowly with position and time on the scales of the environment.

The weak interaction approximation assumes that internal waves are basically a linear phenomenon. The waves propagate along their wave group trajectories, being only slowly modified by dynamical processes. The dynamical evolution of the action density spectrum is then governed by the radiation balance equation.

WORK COMPLETED

(1) Dynamic balance. The dynamic balance determines the internal wave field as a balance of generation, transfer and dissipation processes. There are many dynamical processes that affect internal waves. The first task in designing the internal wave action model is the identification of those dynamical processes that are essential and need to be represented in the model. These processes have been identified, guided by the results of the Eleventh ‘Aha Huliko’a workshop.

(2) Assessment of the accuracy of the WKB approximation. The radiation balance equation is based of the WKB approximation. Its validity and accuracy need to be assessed. One critical area is the scattering of internal waves at finite topography. In two dimensions there exist “exact” analytical and numerical solutions of the wave equation (Müller and Liu, 2000 a, b) that can be used to assess the validity of the WKB approach and to modify it where necessary. In three dimensions such solutions are lacking.

A first step towards deriving “exact” solutions for the scattering of internal waves at finite topography in three dimensions has been completed. Based on the Green’s function of the three-dimensional wave equation a coupled set of integral equations has been derived. The kernels are singular for critical slopes. Applied to the reflection at a straight slope and the scattering at infinitesimal topography, the integral equations reproduce known results.

The integral equations can of course be used to study the scattering at three-dimensional topographic features, not only to determine the range of validity of the WKB approximation.

(3) Effects of the meridional component of the earth’s rotation. The “traditional approximation” neglects the local horizontal component of the earth’s rotation. For low frequency internal waves, one usually employs the smallness of the aspect ration to justify

this approximation (and the hydrostatic approximation). For high frequency waves one expects rotation not to matter at all. There exists, however, no quantitative estimate of the error.

The dispersion relation with and without the meridional component have systematically been compared in mid-latitudes. The difference and its dependence on frequency, wavenumber, latitude, and buoyancy frequency has been determined. A similar comparison for equatorial waves is currently carried out.

(4) Mixing. Breaking of internal waves and subsequent mixing are assumed to be the major processes that drain energy from the internal wave field. In a breaking event, the turbulent eddies are dissipated by molecular viscosity at the same time as they are working against buoyancy forcing. A major question is how much of the internal wave energy is converted to mean potential energy, and how much is dissipated into heat. The ratio is generally thought to be of the order of 0.2, based largely on laboratory measurements. Now, high-resolution numerical simulations of breaking internal waves and the interaction of waves and turbulence are also becoming feasible and informative.

A large eddy simulation (LES) study has been completed to address the interplay of internal waves and mixing in the equatorial surface layer.

RESULTS

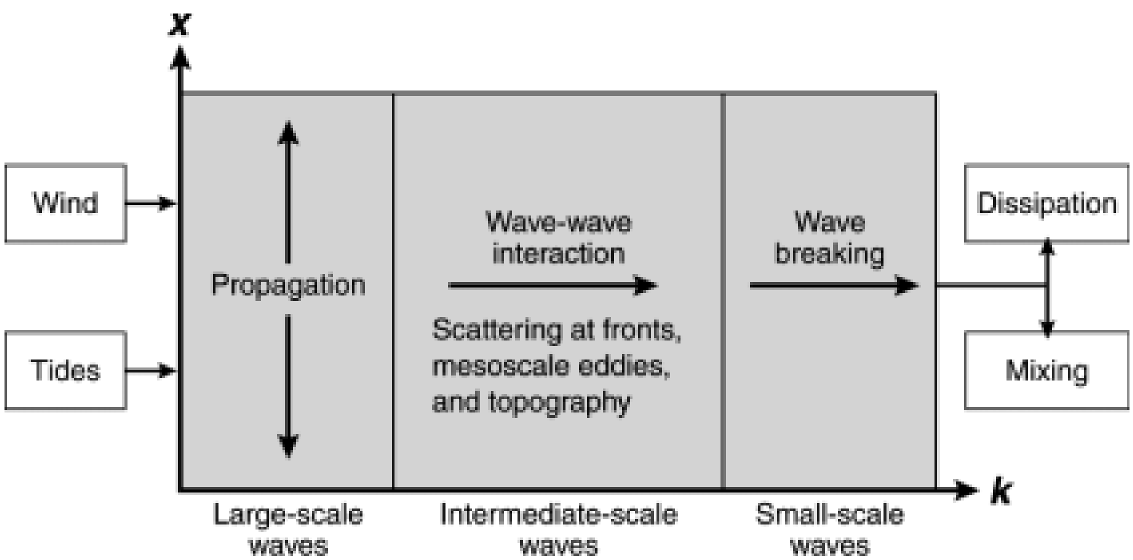

Figure 1. The dynamic balance of the oceanic internal wave field in physical (x) and wavenumber (k) space. The wind and tides generate large-scale waves of near inertial and tidal frequencies. These large-scale waves propagate away from their sources in physical space and cascade towards small-scale waves in wave-number space. The cascade is caused by wave-wave interactions and scattering at fronts, mesoscale eddies, topography, and other scatterers. The small-scale waves break and cause turbulence and mixing

(1) The major dynamical processes that need to be included in the internal wave action model have been identified. Figure 1 show these processes in physical (x) and wavenumber (k) space. These processes are:

- The generation of large scale waves at near-inertial and tidal frequency by the wind and surface tide.
- The propagation in physical space of these large scale waves away from their sources at the surface and bottom.
- The cascade of wave action in wavenumber space from large to small scales by wave-wave interaction and by scattering at fronts, eddies and topography.
- The breaking of small scale waves and the conversion of their energy into mixing and heat.

It might be necessary to include additional processes in later versions of the model, such as the generation of internal waves by the continuous geostrophic adjustment of large scale flows, by surface waves, and by eddies in the mixed layer.

A clear distinction needs to be made between near-inertial internal waves, internal tides and the internal wave continuum.

(2) Using Green's functions, the scattering of internal waves at finite three-dimensional topography has been reduced to solving a set of coupled integral equations. Though the kernels are singular, these equations are amenable to analytic and numerical solutions.

(3) The neglect of the meridional component of the earth's rotation causes an error in the internal wave dispersion relation. In mid-latitudes the maximum relative error is a monotonous function of the parameter $\tilde{f} / N$ where $\tilde{f}$ is twice the meridional component of the earth's rotation and N the buoyancy frequency. It is largest for small N and low latitudes. It is order one for $\tilde{f} / N = 0$ (1).

(4) The Large-Eddy Simulation (LES) study confirmed that internal waves are an integral part of mixing and deep cycle turbulence in the equatorial mixed layer.

IMPACT/APPLICATION

The development of a predictive dynamical model of the global or regional internal wave fields will have many benefits and applications.

Internal wave research will benefit from such a model since

- it will provide understanding of the internal wave field as a balance of generation, transfer and dissipation processes,
- it will focus research (it is expected that the proposed model will do for internal wave dynamics what the GM model did for internal wave kinematics), and

- it will predict changes of the internal wave field in response to changes in the forcing and environmental fields.

The dynamical internal wave model can be run in conjunction with circulation models, turbulence models, chemical tracer models, and biological population models where it would predict the internal wave induced transports, dispersion and mixing. In conjunction with acoustic transmission models the model would predict the internal wave induced "noise."

TRANSITIONS

RELATED PROJECTS

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